神奈川県立産業技術総合研究所
2017年9月12日

計算力学の基礎

－有限要素法を中心として－

法政大学 デザイン工学部

竹内　則雄

HOSEI

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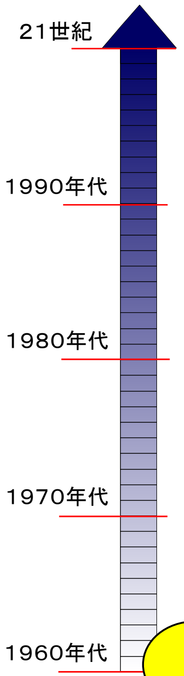
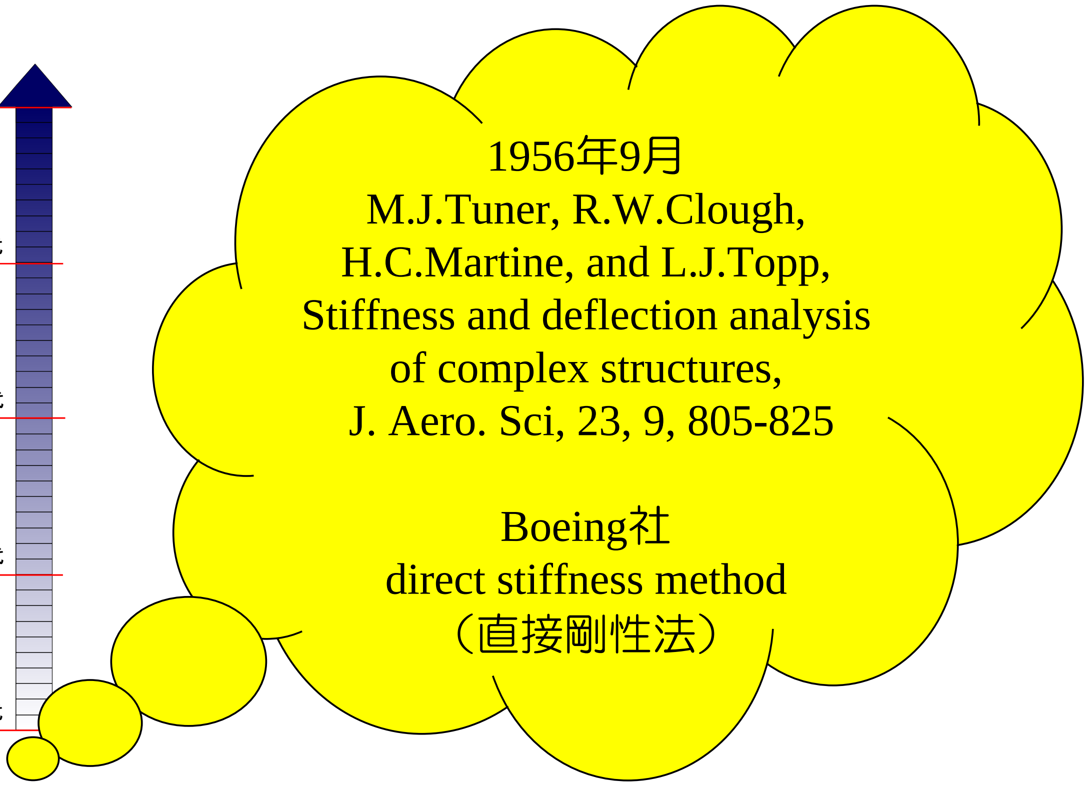
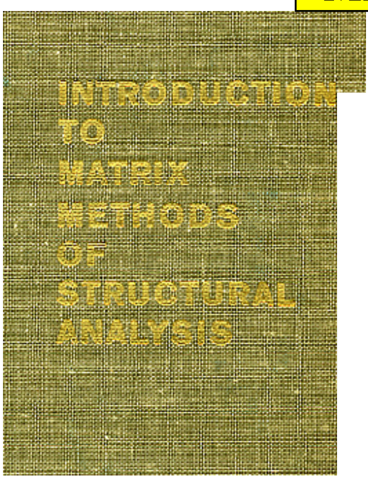
HOSEI

FEM創成期





THE FINITE-ELEMENT MATRIX METHODS (4P)



1966年出版

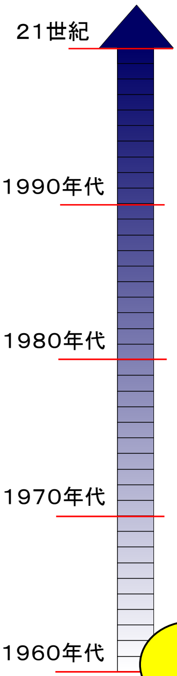
「マトリックス法による構造力学の解法」（培風館 1967）

INTRODUCTION TO MATRIX METHODS OF STRUCTURAL ANALYSIS

HAROLD C. MARTIN
Professor of Aeronautics and Astronautics, University of Washington, and Consultant, Aerospace Group, The Boeing Company, Seattle, Washington

McGRAW-HILL BOOK COMPANY
New York St. Louis San Francisco Toronto London Sydney

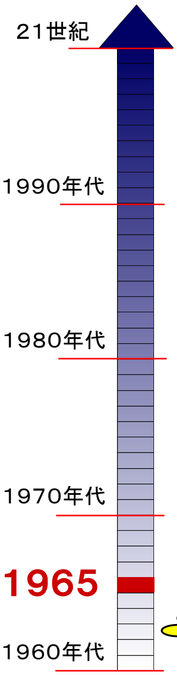
FEM創成期



1960年
J.H.Argyris and S. Kelsey,
Energy theorems and
structural analysis,
Butterworth, London

エネルギー原理に基礎をおく
マトリックス構造理論
の一般形による公式化
応力法に重点が置かれているが
内力に代わり変位を未知量に
することも述べている

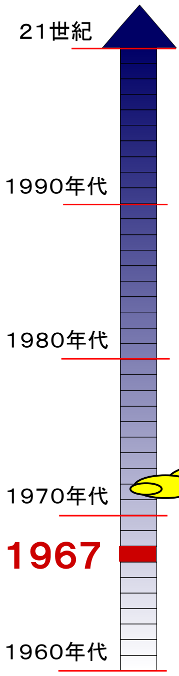
日本におけるFEM研究の夜明け

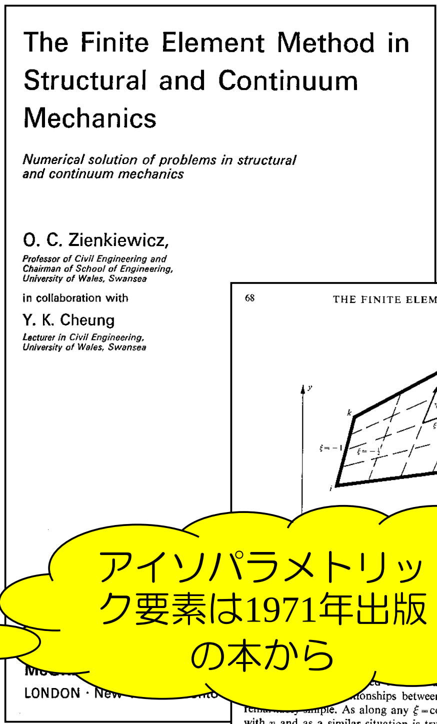


1965年
日本鋼構造協会構造解析小委員会設置
月例会を開催
初代委員長　吉識雅夫　東大教授(当時)
鷲津久一郎・山本善之・川井忠彦・宮本博・山田嘉昭

鷲津・山本・川井先生が
学内で勉強会を開催

発展





The Finite Element Method in Structural and Continuum Mechanics

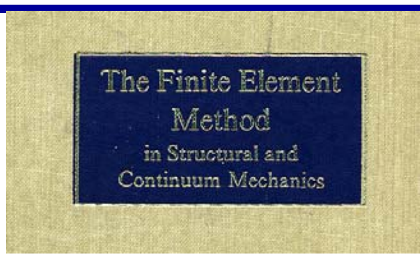
Numerical solution of problems in structural and continuum mechanics

O. C. Zienkiewicz,
Professor of Civil Engineering and Chairman of School of Engineering, University of Wales, Swansea

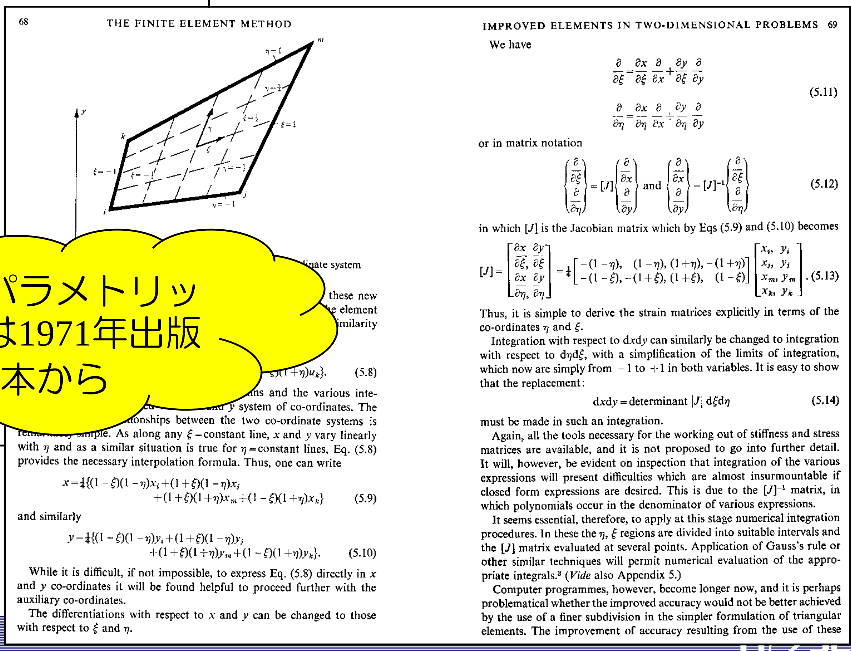
in collaboration with

Y. K. Cheung
Lecturer in Civil Engineering, University of Wales, Swansea

LONDON · New



1967年出版



68 THE FINITE ELEMENT METHOD

with η and as a similar situation is true for η = constant lines, Eq. (5.8) provides the necessary interpolation formula. Thus, one can write

$$x=\tfrac{1}{4}\{(1-\xi)(1-\eta)x_i+(1+\xi)(1-\eta)x_j+(1+\xi)(1+\eta)x_m+(1-\xi)(1+\eta)x_k\} \quad (5.9)$$

and similarly

$$y=\tfrac{1}{4}\{(1-\xi)(1-\eta)y_i+(1+\xi)(1-\eta)y_j+(1+\xi)(1+\eta)y_m+(1-\xi)(1+\eta)y_k\}. \quad (5.10)$$

While it is difficult, if not impossible, to express Eq. (5.8) directly in x and y co-ordinates it will be found helpful to proceed further with the auxiliary co-ordinates.

The differentiations with respect to x and y can be changed to those with respect to ξ and η.

IMPROVED ELEMENTS IN TWO-DIMENSIONAL PROBLEMS 69

We have

$$\frac{\partial}{\partial\xi}=\frac{\partial x}{\partial\xi}\frac{\partial}{\partial x}+\frac{\partial y}{\partial\xi}\frac{\partial}{\partial y}$$
$$\frac{\partial}{\partial\eta}=\frac{\partial x}{\partial\eta}\frac{\partial}{\partial x}+\frac{\partial y}{\partial\eta}\frac{\partial}{\partial y} \quad (5.11)$$

or in matrix notation

$$\begin{Bmatrix}\frac{\partial}{\partial\xi}\\ \frac{\partial}{\partial\eta}\end{Bmatrix}=[J]\begin{Bmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\end{Bmatrix} \text{ and } \begin{Bmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\end{Bmatrix}=[J]^{-1}\begin{Bmatrix}\frac{\partial}{\partial\xi}\\ \frac{\partial}{\partial\eta}\end{Bmatrix} \quad (5.12)$$

in which $[J]$ is the Jacobian matrix which by Eqs (5.9) and (5.10) becomes

$$[J]=\begin{bmatrix}\frac{\partial x}{\partial\xi}, & \frac{\partial y}{\partial\xi}\\ \frac{\partial x}{\partial\eta}, & \frac{\partial y}{\partial\eta}\end{bmatrix}=\tfrac{1}{4}\begin{bmatrix}-(1-\eta), & (1-\eta), & (1+\eta), & -(1+\eta)\\ -(1-\xi), & -(1+\xi), & (1+\xi), & (1-\xi)\end{bmatrix}\begin{bmatrix}x_i, & y_i\\ x_j, & y_j\\ x_m, & y_m\\ x_k, & y_k\end{bmatrix}. \quad (5.13)$$

Thus, it is simple to derive the strain matrices explicitly in terms of the co-ordinates η and ξ.

Integration with respect to $dxdy$ can similarly be changed to integration with respect to $d\eta d\xi$, with a simplification of the limits of integration, which now are simply from -1 to $+1$ in both variables. It is easy to show that the replacement:

$$dxdy = \text{determinant } |J|\, d\xi d\eta \quad (5.14)$$

must be made in such an integration.

Again, all the tools necessary for the working out of stiffness and stress matrices are available, and it is not proposed to go into further detail. It will, however, be evident on inspection that integration of the various expressions will present difficulties which are almost insurmountable if closed form expressions are desired. This is due to the $[J]^{-1}$ matrix, in which polynomials occur in the denominator of various expressions.

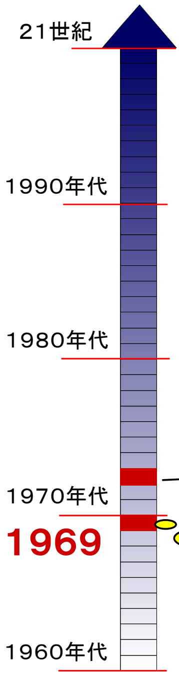
It seems essential, therefore, to apply at this stage numerical integration procedures. In these the η, ξ regions are divided into suitable intervals and the $[J]$ matrix evaluated at several points. Application of Gauss's rule or other similar techniques will permit numerical evaluation of the appropriate integrals.[3] (*Vide* also Appendix 5.)

Computer programmes, however, become longer now, and it is perhaps problematical whether the improved accuracy would not be better achieved by the use of a finer subdivision in the simpler formulation of triangular elements. The improvement of accuracy resulting from the use of these

アイソパラメトリック要素は1971年出版の本から

HOSEI

第１回日米セミナー開催



１９６９年８月東京
第１回マトリックス構造解析に関する日米セミナー開催
（日本学術会議・米国国立科学財団）
アメリカ側代表：Prof. R.H.Gallagher (Cornell Univ.)
米国側参加者　約３０名
日本側参加者　約１００名

１９７２年８月Berkeley
第２回日米セミナー
on Matrix Methods of
Structural Analysis and Design

アポロ１１号
月面着陸

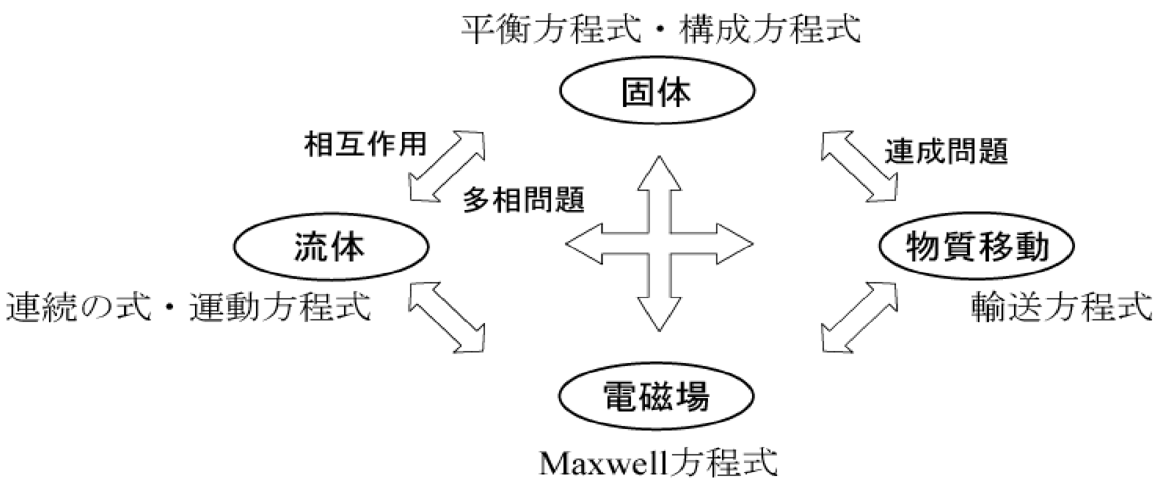
HOSEI

現象のモデル化と離散化手法

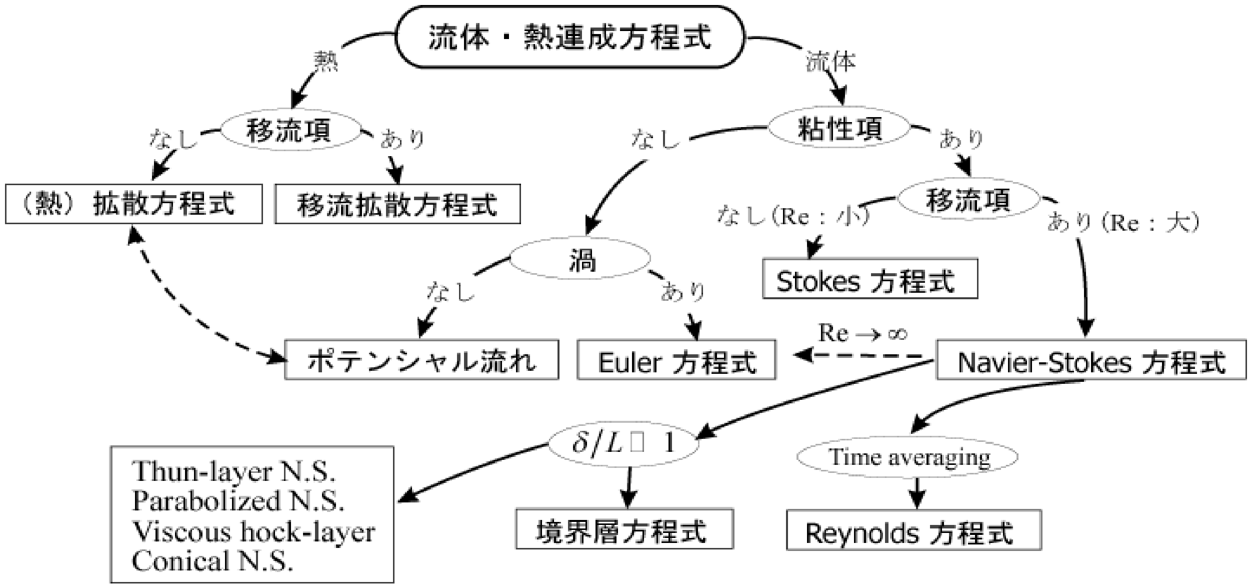
学習目標

物理現象の数理モデル化の考え方とその離散化手法について概念を学ぶ

多重複合物理現象の理論的背景と代表的偏微分方程式



流体と熱の支配方程式



流体力学における基礎方程式(ニュートン流体)

(連続の式) $\dfrac{D\rho}{Dt} + \rho \dfrac{\partial v_i}{\partial x_j} = 0$ $\qquad \dfrac{D\rho}{Dt} + \rho \nabla \cdot \boldsymbol{v} = 0$

(運動方程式) $\rho \left(\dfrac{\partial v_i}{\partial t} + v_j \dfrac{\partial v_i}{\partial x_j} \right) = \dfrac{\partial \sigma_{ij}}{\partial x_j} + \rho f_i$ $\qquad \rho \left(\dfrac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} \right) = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{f}$

(構成方程式) $\sigma_{ij} = -p\delta_{ij} + \mu \left[\left(\dfrac{\partial v_i}{\partial x_j} + \dfrac{\partial v_j}{\partial x_i} \right) - \dfrac{2}{3} \delta_{ij} \dfrac{\partial v_k}{\partial x_k} \right]$

$$\boldsymbol{\sigma} = -\nabla p - \mu \nabla^2 \boldsymbol{v} - (\lambda + \mu) \nabla (\nabla \cdot \boldsymbol{v})$$

・Navier-Stokes方程式

$$\rho \left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = \rho f_i - \frac{\partial p}{\partial x_i} + \mu \left[\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial v_k}{\partial x_k} \right] \qquad \rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} \right) = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{f}$$

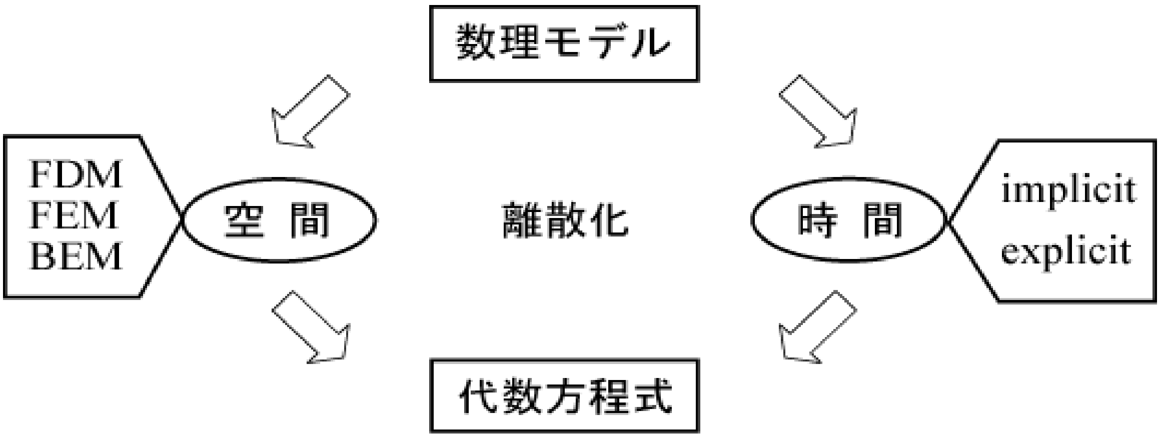
・Euler方程式(粘性を無視した場合)

$$\rho \left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = \rho f_i - \frac{\partial p}{\partial x_i} \qquad \rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} \right) = -\nabla p + \rho \boldsymbol{f}$$

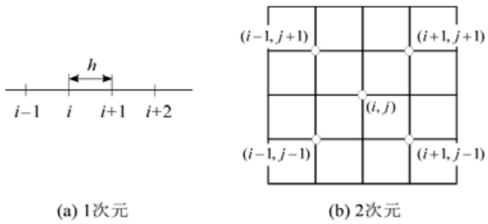
・Stokes方程式(遅い流れの場合)

$$\rho \frac{\partial v_i}{\partial t} = \rho f_i - \frac{\partial p}{\partial x_i} + \mu \left[\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial v_k}{\partial x_k} \right] \qquad \rho \frac{\partial \boldsymbol{v}}{\partial t} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{f}$$

離散化手法



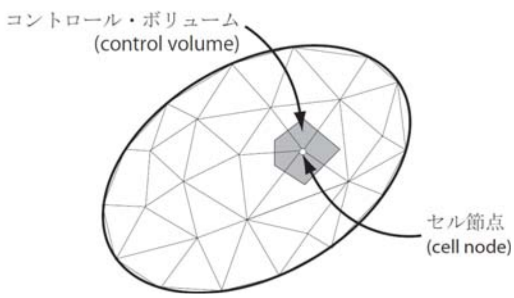
各種解析法の領域分割



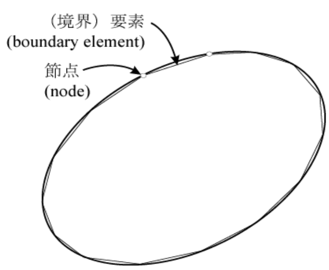
$$\frac{du}{dx} = \lim_{h\to 0}\frac{u(x+h)-u(x)}{h} \approx \frac{u(x+h)-u(x)}{h}$$

$$\frac{d^2u}{dx^2} \approx \frac{\dfrac{u_{i+2}-u_{i+1}}{h}-\dfrac{u_{i+1}-u_i}{h}}{h} = \frac{u_{i+2}-2u_{i+1}+u_i}{h^2}$$

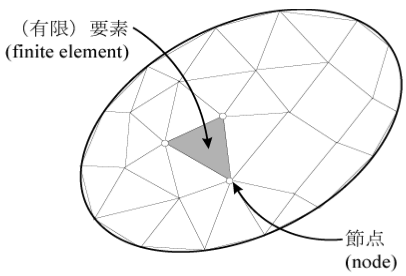
FDM (Finite Difference Method)



FVM (Finite Volume Method)

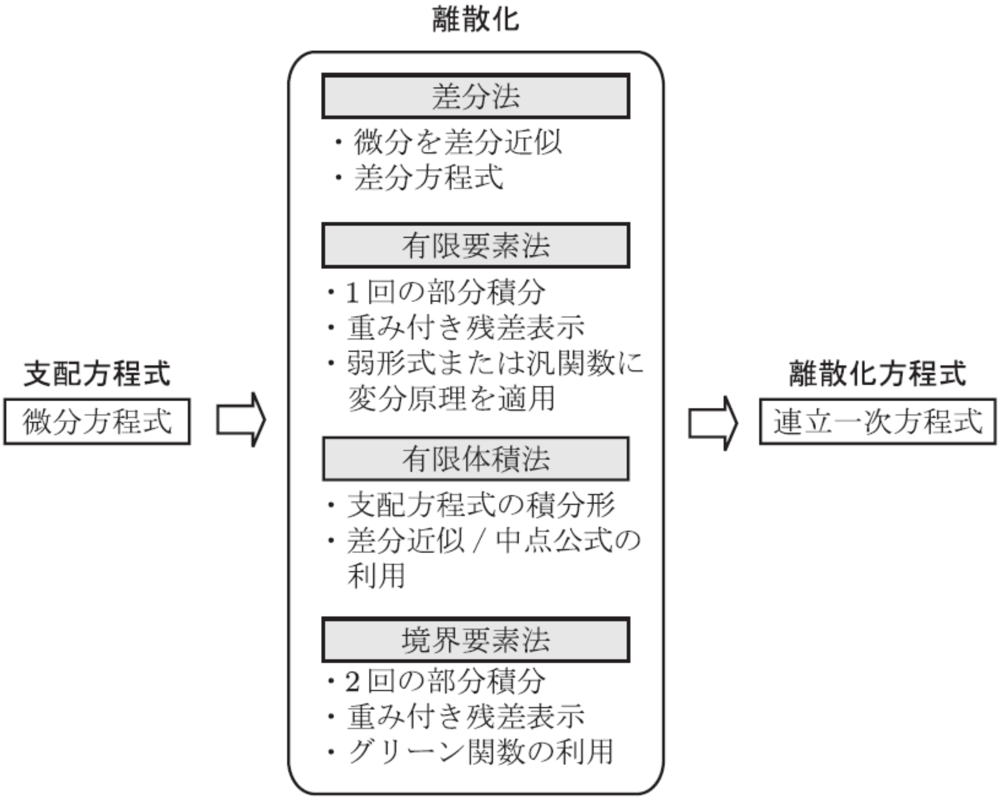


BEM (Boundary Element Method)



FEM (Finite Element Method)

離散化の手順と関係



解析の手順